International Journal of Theoretical Physics, Vol. 33, No. 11, 1994

Condensed Matter Effects in the Solar Neutrino Problem

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Received January 25, 1994

In addition to our earlier corrections to fusion cross sections, we proposed that previously overlooked condensed matter effects (CME) can help to account for the missing solar neutrino flux. There are three important CME. One is due to a reduction in collision frequency due to an exchange of kinetic and potential energies in collision processes. Another is an excluded volume effect. The third is a shadowing effect due to the presence of spectator species which do not participate in fusion. These CME become appreciable in the high densities encountered in stellar media where they significantly affect fusion rates, since the solar core plasma cannot accurately be described as a collisionless ideal gas. Contrary to Bahcall and Gould (1993), we do not violate Liouville's theorem, the Maxwellian distribution, nor thermodynamics in our proposed solution to the solar neutrino problem.

1. INTRODUCTION

It is intriguing that the enigmatic solar neutrino puzzle may have a solution in careful consideration of the details rather than with grand new ideas. The standard solar model (SSM) has been successful in relating the mass and composition of the sun to its luminosity and lifetime. The SSM has also been widely accepted, as it appears to be based upon well-understood nuclear physics. However, as we have shown, this has included cross-section approximations that work well at high energies, but are invalid for the solar energy regime (Kim *et al*, 1993*c*). In fact the SSM has appeared to work so well that the preponderance of attempted theoretical solutions has been directed at the neutrinos, rather than the SSM. A wide range of imaginative proposed solutions abound, of which oscillation of the electron neutrino to another neutrino family has been one of the more

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0020-7748/94/1100-2189$07.00/0

popular. This has yet to be demonstrated, and depends upon a nonzero neutrino rest mass, which is also speculative.

While accepting the basic framework of the SSM, we find that significantly different predictions result by including overlooked condensed matter effects (CME) as input into the SSM (Kim *et al.*, 1993*a*,*b*). The virial equation for the dense solar core plasma deviates significantly from that of a collisionless ideal gas espoused by Bahcall and Gould (1994) (BG), as shown by a recent molecular dynamics simulation (Braswell and Kim, 1993*a*,*b*; Kim *et al.* 1993*d*, 1994). In dense stellar plasmas, the ensemble of fusing particles converts a significant fraction of the kinetic energy into potential energy, thus diminishing the flux and hence collision rate. We call this the condensed matter energy exchange effect (CMEE). The flux is selectively reduced much more for the $^7Be(p, \gamma)^8B$ reaction, because of the greater Coulomb barrier, than for the $p(p, e^+\nu_e)D$ reaction. This is just what is needed, as the latter reaction rate is fairly well established, whereas the former is thought to be higher than experimentally measured by as much as a factor of three. In the presence of nonfusing spectator species, there is also a high-density condensed matter interference effect (CMIE). CMIE may be thought of as due to shadowing or interference caused by spectator species, resulting in reduced fusion rates. An additional CME is related to excluded volume (CMEVE), which has an analog in liquids and inside nuclei (Kim *et al.*, 1993*a*,*b*,*d*, 1994).

In some cases, even at high density, the noninteraction collisionless models as used by BG can yield accurate predictions. This may be because some of the CME act to decrease fusion rates, whereas others serve to increase fusion rates. In the CMEE, the conversion of kinetic to potential energy at high densities decreases fusion rates, as does the spectator interference effect (CMIE). On the other hand, for a given temperature the excluded volume effect increases the collision frequency and hence increases fusion rates. The presence of spectator species also acts to increase CMEVE. However, one cannot always count on a cancellation of opposing effects, and it is necessary to use a more sophisticated, albeit more complex model including CME.

CME affect fusion rates appreciably only at the high densities encountered in stellar media, and quickly diminish at ordinary densities. However, we purposely have called these effects condensed matter effects rather than high-density effects because in collisionless models such as that of BG it makes no difference whether the density is high or low. In a noninteracting gas the collisionless calculations of BG would be valid at all densities. But they are not valid at high densities when CME are present and do not cancel out. Recent molecular dynamics simulation studies (Braswell and Kim, 1993, n.d.; Kim *et al.*, 1993*d*, 1994) indicate that CMEE and CMEVE are more significant at lower energies than the 1.2 keV typical of the solar interior, and/or at higher densities than the solar core.

2. CONDENSED MATTER ENERGY EXCHANGE EFFECT (CMEE)

Although we have examined three CME (Kim *et al.*, 1993*a*,*b*,*d*, 1994), Bahcall and Gould (1993) have chosen to focus only upon our CMEE. They say, "We show with the aid of Liouville's theorem that the effects in question are already included in the standard treatment [p. 1] . . . the standard treatment implicitly assumes that nuclei are collisionless [p. 3]." The conventional standard treatment assumed by BG that nuclei are collisionless leads to an inaccurate description of the dense solar core plasma as shown by molecular dynamics simulation (Braswell and Kim, 1993*a*,*b*; Kim *et al.*, 1993*d*, 1994), and hence CMEE and other CME are needed to improve estimates of collision and fusion rates.

Our main premise with respect to CMEE is that it significantly decreases the collision rate for all collisions at solar core densities, and in particular the subset of those collisions that lead to fusion. It is noteworthy that BG do not address the "collision rate" issue in their rebuttal endeavor. This is strange since it is at the heart of our analysis, and should be the key point of contention if they truly have a valid point of disagreement.

In a dense plasma such as the solar interior, because the time average of the potential energy is a significant component of the total energy, the time-averaged kinetic energy is reduced compared to a collisionless gas of the same total energy. This reduces the effective flux of fusing nuclei in the fusion rate formula, their total collision rates, and consequently their fusion rates. The nuclei slow down as they approach the repulsive shielded Coulomb barrier. Thus the initial velocity or asymptotic velocity v, as nuclei are far apart, needs to be replaced by a time average velocity u in fusion rate calculations. At ordinary densities or at very high energies, the difference between v and u is negligible, but not at the keV energy and 10^2 gm/cm^3 density in the solar core, as we have shown (Kim *et al.*, 1993*a*,*b*).

A concomitant effect is the reduction in density in the neighborhood of collision points. This is a local effect near collision centers which has an analog to the global decrease in density in the earth's atmosphere with increasing altitude due to the earth's gravitational potential. The locally decreased velocities and decreased densities are consistent with both the Maxwellian distribution and Liouville's theorem. Although the local distribution function for microcanonical ensembles in the neighborhood of collision points does not have to be the same as the overall global distribution, we conservatively used the same Maxwellian distribution here. Liouville's theorem teaches the conservation of the density of possible points of a system in phase space. Thus if the velocity component of phase space decreases, the spatial component increases. For a system with a fixed number of particles, the increase in the spatial component implies a decrease in number density (number of nuclei/volume) in real space. Thus we do not claim anything that is contrary to the overall Maxwellian distribution or to Liouville's theorem.

Aside from a direct quotation of ours, which is correctly presented, BG often misrepresent our arguments. For example, they say, "Specifically, they argue that in the dense environment of the Sun, the phase-space densities of fusing nuclei are lower at the point where they last scatter off ambient particles (prior to reacting) than are the phase-space densities of the same particles when they are far apart" (Bahcall and Gould, 1994, p. 995). We did not say this and have always been aware that Liouville's theorem conserves the phase-space density. We have always dealt with the "real-space" density. We showed in the preceding paragraph that Liouville's theorem implies that the real-space density decreases. The real-space density may be obtained by integrating the phase-space density with respect to the velocity components. Nevertheless, the two densities are quite different.

In our previous work (Kim *et al.*, 1993*a*,*b*), we showed that the Maxwellian distribution also implies a decrease in the real-space density. Even though the Maxwellian is unchanged, the overall normalization is changed because of the exponential factor containing the potential energy term.

The solar core density of $\sim 30/\text{Å}^3$ corresponds to an average center-to-center spacing of ions of ~ 0.32 Å. Since the screening radius is $a \approx 0.25$ Å (Salpeter, 1954) for the Debye–Hückel potential $V(r) = Z_1 Z_2 e^2 e^{-r/a}/r$, the solar core (proton–electron) plasma cannot properly be described as a collisionless ideal gas. There is no region in the solar core where the potential energy vanishes. The potential energy at the equilibrium radius r_e is not the same as the zero of potential energy at infinity. Neither do we advocate that "Hence the calculation of reaction rates (which implicitly assumes free particles) should use the phase-space distribution of particles at r_e rather than at infinity" as incorrectly ascribed to us by Bahcall and Gould (1994, p. 996).

BG improperly attempted to relate, via Liouville's theorem, two distinctly different physical systems that have different Hamiltonians. One is the real nonideal gas (dense solar core plasma), and the other is a collisionless ideal gas. Although the latter is the low-density limit or noninteraction limit of the former, the two are not physically the same. Liouville's theorem applies to either one individually, but not to both the nonideal gas and the ideal gas simultaneously and indiscriminantly. Of course we think that only a real gas calculation should be used in the SSM, just as real gas van der Waals or virial correlations are standardly used when more accurate results are needed than given by the ideal gas law.

3. FLUCTUATIONS

BG would have the reader think that not only do we transgress Liouville's theorem, but that we also espouse an even more drastic violation of the laws of physics. We made a parenthetical remark that there may be a departure from thermal equilibrium "locally" as kinetic energy is converted to potential energy during collisions. This leads BG to conclude, "However, such a departure from equilibrium would require a violation of the Second Law of Thermodynamics" (Bahcall and Gould, 1994, p. 998). Fluctuations are a deviation from thermal equilibrium and are caused by collisions. Fluctuations are an accepted part of statistical mechanics and thermodynamics, and are compatible with the second law. It is well known that even in those instances when fluctuations lead to a decrease in entropy locally, the total entropy still increases globally.

4. CONDENSED MATTER EXCLUDED VOLUME EFFECT (CMEVE)

It is well established that for liquid and dense gaseous systems, the molecular collision rate is significantly different from the value predicted by the simple kinetic theory championed by BG, due to the excluded volume effect (Hansen and McDonald, 1986). The excluded volume effect is important in nuclear theory, and significantly increases the Fermi energy of nucleons in a nucleus (Weisskopf, 1972). We have investigated CMEVE in the solar core by a molecular dynamics simulation (Kim *et al.*, 1993*d*, 1994).

From kinetic theory, the collision rate for particles in an ideal gas is given by

$$\Gamma_0 = \sqrt{2} n \sigma u \tag{1}$$

where n is the number density, σ is the collision cross section, and u is the mean speed. Hansen and McDonald (1986) show that relaxing the ideal gas assumption yields for a hard-sphere system a collision rate as given by

$$\Gamma = \Gamma_0 g(d) \tag{2}$$

where $g(d)$ is the radial distribution function $g(r)$ evaluated at the hard-sphere diameter $r = d$. The radial distribution function is defined (Allen and Tildesley, 1987) as the number of particles, on average, in the volume $4\pi r^2\, dr$ centered about a given particle divided by the number that would be in the same volume if the system behaved as an ideal gas. Hence for an ideal gas $g(r) = 1$.

Braswell and Kim (1993*a,b*) used molecular dynamics to compute the radial distribution function for nuclei with a screened Coulomb potential (Salpeter, 1954) in a simulation of the solar core. At a temperature of 1.2 keV and below, there was a significant deviation from an ideal gas (Kim *et al.*, 1994). The results of $g(r) < 1$ at short distances support CMEE for the solar neutrino problem as previously calculated (Kim *et al.*, 1993*a,b*).

For a given radial distribution $g(r)$, the ratio p/nkT can be calculated from the virial (or pressure) equation (Hansen and McDonald, 1986). This is given by

$$\frac{p}{nkT} = 1 - \frac{n}{6kT} \int_0^\infty V'(r) r g(r) \, d^3r \tag{3}$$

where $V'(r) = dV(r)/dr$. The second term on the right side of equation (3) is the nonideal contribution. With $g(r)$ computed from a molecular dynamics simulation (Braswell and Kim, 1993, n.d.) using the Debye–Hückel potential (Salpeter, 1954) for the proton–electron solar core plasma, the virial equation becomes

$$\frac{p}{nkT} = 1 + \left(\frac{2\pi}{3}\right) \frac{ne^3}{kT} (3a^2)(0.994) = 1.143 \tag{4}$$

As input to equation (4), we used the solar core density of $30.12/\text{Å}^3$, the screening radius $a = 0.251$ Å, and the solar core temperature $kT = 1.2$ keV.

If we wish to describe the above result for the solar core plasma in terms of an ideal gas formulation for a fictitious collisionless gas with the same density and temperature as BG do, equation (4) may be rewritten in the form of an ideal gas

$$\frac{p^*}{nkT} = 1 \tag{5}$$

where $p^* = p/1.143$ is an effective (fictitious) pressure for the collisionless ideal gas model which BG and others use. Having the effective pressure p^* less than the real pressure p indicates the effects of a repulsive potential in both the virial of Clausius and van der Waals formulations of real gases. This justifies our use of CME, contrary to the claims of BG.

Insofar as the idealized collisionless point model of BG goes, as constrained by the conservation of energy, the spatial component in phase space may become arbitrarily small in any and all dimensions as long as the overall phase-space volume for their system remains constant. This is unphysical, as within the constraints of the given gravitational potential, the ions and their shielded Coulomb potentials cannot all occupy the same space. This implies that there is a minimum spatial volume and minimum coordinate range that a real system can attain in phase space consistent with energy conservation. Thus CMEVE must be included as a boundary condition even in the Liouville approach to obtain realistic results for a dense system.

5. CONDENSED MATTER INTERFERENCE EFFECT (CMIE)

The presence of nonfusing spectator species leads to CMIE, i.e., a shadowing effect in which spectator species get in the way of collisions between fusing species. We presented a calculation for CMIE (Kim *et al.*, 1993*a*) representative of conditions in the solar core. This showed that the presence of species like ^{4}He has a nonnegligible effect in reducing fusion rates.

Again the idealized collisionless point model of BG is unphysical with respect to the two roles spectator species play at high densities. For BG, spectator species cannot interfere in preventing collisions between fusing nuclei, as they have no volume. Because of their volume, spectator species also act to limit the accessible regions of phase space. Thus the neglect of both CMIE and CMEVE in the collisionless BG approach can lead to unrealistic results for highly dense sysems containing an appreciable fraction of spectator species such as ^{4}He in the solar interior.

6. LIOUVILLE'S THEOREM

Our CME do not violate Liouville's theorem. However, because this theorem is key to the work of BG let us examine it briefly. In the absence of collisions, the Boltzmann transport equation can yield Liouville's theorem. Liouville (1837) showed that the measure of a set of points is an invariant of the natural motion in phase space. In other words, if any portion of phase space is densely and uniformly filled with moving points representing a dynamical system in different possible states of motion, then the laws of motion are such that the density of these points remains constant in phase space. Each system is represented by a single point in phase space, and the ensemble of systems corresponds to a swarm of points in phase space. Thus each point represents a system of N particles, where $N \sim 10^{23}$ in a phase space of $3N$ spatial coordinates and $3N$ momentum coordinates. All the members of the ensemble are as like our system of interest as permitted by physics and our knowledge, but they may have any of the initial conditions that are allowable. The many replicas of the system of interest occupy all the admissible initial conditions and times. The volume of phase space between two energy surfaces E and $E + \Delta E$ has the physical dimensions of (energy $\times$ time)^{3N}.

Since Liouville's theorem is considered so central by some in the methodology related to the solar neutrino problem as well as plasma fusion in general, let us consider its applicability and limitations. First of all it only applies in a nondissipative system in which energy (KE + PE) is conserved. Strictly speaking, the solar energy is not conserved. Second, realization of Liouville's theorem depends subtlely on the ergodic hypothesis, since it is assumed that the system can indeed move from one region of phase space to another based upon the equations of motion, consistent with the conservation of energy. The ergodic hypothesis implies that every state of the system can be reached directly or indirectly from every other state. This means that if the energy of the system is determined within a range ΔE, the probability of finding the system in a certain state compatible with that energy is the same for each state. This is the basis of statistical mechanics, wherein the time average over the evolution of the system is replaced by the average over the different states.

"It can actually be demonstrated that a classical system cannot be truly ergodic" (Mayer and Mayer, 1940). Hence the work of BG cannot be strictly rigorous, even without CME considerations, because their implementation of Liouville's theorem depends upon the ergodic hypothesis. In a system such as the sun, where fusion is occurring, we do not think that Liouville's theorem applies rigorously even for quantum mechanics due to the disappearance of high-energy states for given species in the act of fusion. At predominantly high energies, the fusing species disappear and new species are formed. Our work respects Liouville's theorem and does not trespass it. Nevertheless, we point out these relatively minor things to show that Liouville's theorem should be used with care in such environments.

7. SUMMARY AND CONCLUSIONS

We presented three CME and an analysis showing a significant error in the cross sections used in the standard solar model. BG have chosen only to address our condensed matter energy exchange effect (CMEE). Since they have ignored the others, is this an indication that this is the only one of our four points with which they disagree? With respect to our CMEE, BG have not addressed our main point that the collision and hence fusion rates are decreased. Rather they have raised a bogus issue related to Liouville's theorem and phase space rather than real space.

Our CMEE relates to a decrease in collision and hence fusion rates because of a slowing down of the fusing nuclei as kinetic energy is converted to potential energy. This also involves a reduction in real-space number density in the neighborhood of colliding nuclei as they approach the classical turning point compared with their real-space number density when they are far apart. The real-space density is *not* the same as the phase-space density, though BG seem to think they are the same.

According to Liouville's theorem, the phase-space density should be conserved at all points in phase space. Even if the phase-space density is conserved, the real-space number density decreases, decreasing the number of nuclei/volume, for a fixed number of particles because the spatial component increases as the velocity space decreases. The real-space number density decreases as the nuclei convert kinetic energy to potential energy, much the same as the earth's atmospheric density decreases as the gravitational potential increases. In the case of the atmosphere there is also a reduction in the temperature at the highest altitudes with concomitantly low density so that the collision frequency is insufficient to restore the temperature corresponding to lower altitudes. In a gas or plasma, a collision is essentially the conversion of kinetic to potential energy and back again to kinetic energy. Fluctuations occur during collisions in which small local regions in the neighborhood of the collisions deviate from thermal equilibrium but do not violate the second law of thermodynamics. Temperature and density fluctuate locally in time and space, which is a more complicated problem than just the analog to the increased gravitational potential in the earth's atmosphere.

BG not only misrepresent our views, they are also not consistent in presenting their own views. For example BG say, "Here we show that the reduced phase space density at close separations is *already* taken into account in the standard calculations" (Bahcall and Gould, 1994, p. 995). By Liouville's theorem, the phase-space density is conserved everywhere. Hence, contradictory to BG, the phase-space density is *not* "reduced" at close separations. Furthermore, contrary to the views of BG, we do not violate Liouville's theorem, the Maxwellian distribution, nor thermodynamics in our proposed solution to the solar neutrino problem. We do show that our three CME have a significant influence on fusion rates, as do the corrected fusion cross sections. These considerations should be included in the input to the standard solar model and stellar model calculations, just as virial or van der Waals corrections are made to real gases. Our CME provide estimates of corrections for the collision and fusion rates for the nonideal gas contributions in the solar core. As an alternative approach we have also applied virial corrections to the solar core and obtain results consistent with our earlier calculations that the collision frequency and fusion rates are significantly reduced.

ACKNOWLEDGMENTS

We wish to thank Arthur Cohn, Mark Davidson, David Fryberger, Paul Muzikar, and Sergio Rodriquez for their interest and helpful discussions.

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